Pion photoproduction in a gauge invariant approach

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Introduction

- Study of N^* is important for understanding of strong interactions

  A clear understanding of N^* structure, spectrum and decay will reveal the role of confinement and chiral symmetry in QCD non-perturbative region.

- Dynamical coupled-channel models are needed

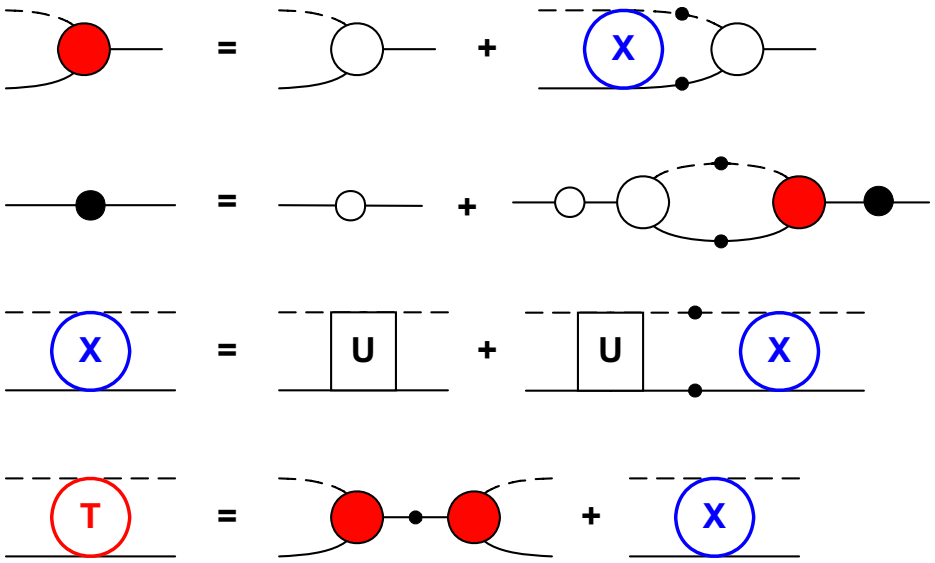
  N^* states are unstable and couple strongly with meson-baryon continuum states. In order to extract N^* parameters and understand N^* structures, dynamical coupled-channel models are needed to analyze the meson production data.

- Gauge invariance is important for photo-production

  Gauge invariance is a fundamental symmetry. Without it results become arbitrary. Note current conservation is necessary but not sufficient for gauge invariance.

- This work: gauge invariant approach for π photo-production in Jülich dynamical coupled-channel model

Hadronic scattering



$$|\Gamma\rangle = |F\rangle + X\,G\,|F\rangle$$

$$S = S_0 + S_0\,\langle F|\,G\,|\Gamma\rangle\,S$$

$$X = U + U\,G\,X$$

$$T = |\Gamma\rangle\,S\,\langle\Gamma| + X$$

$|F\rangle$: bare vertex

S_0 : bare propagator

X : non-polar part of T matrix

T : full T matrix

$|\Gamma\rangle$: dressed vertex

S : dressed propagator

U : driving term of X

Jülich πN meson-exchange model

Dynamical coupled-channel model: $N\pi$, $N\eta$, $\Delta\pi$, $N\rho$, $N\sigma$, ΛK, ΣK

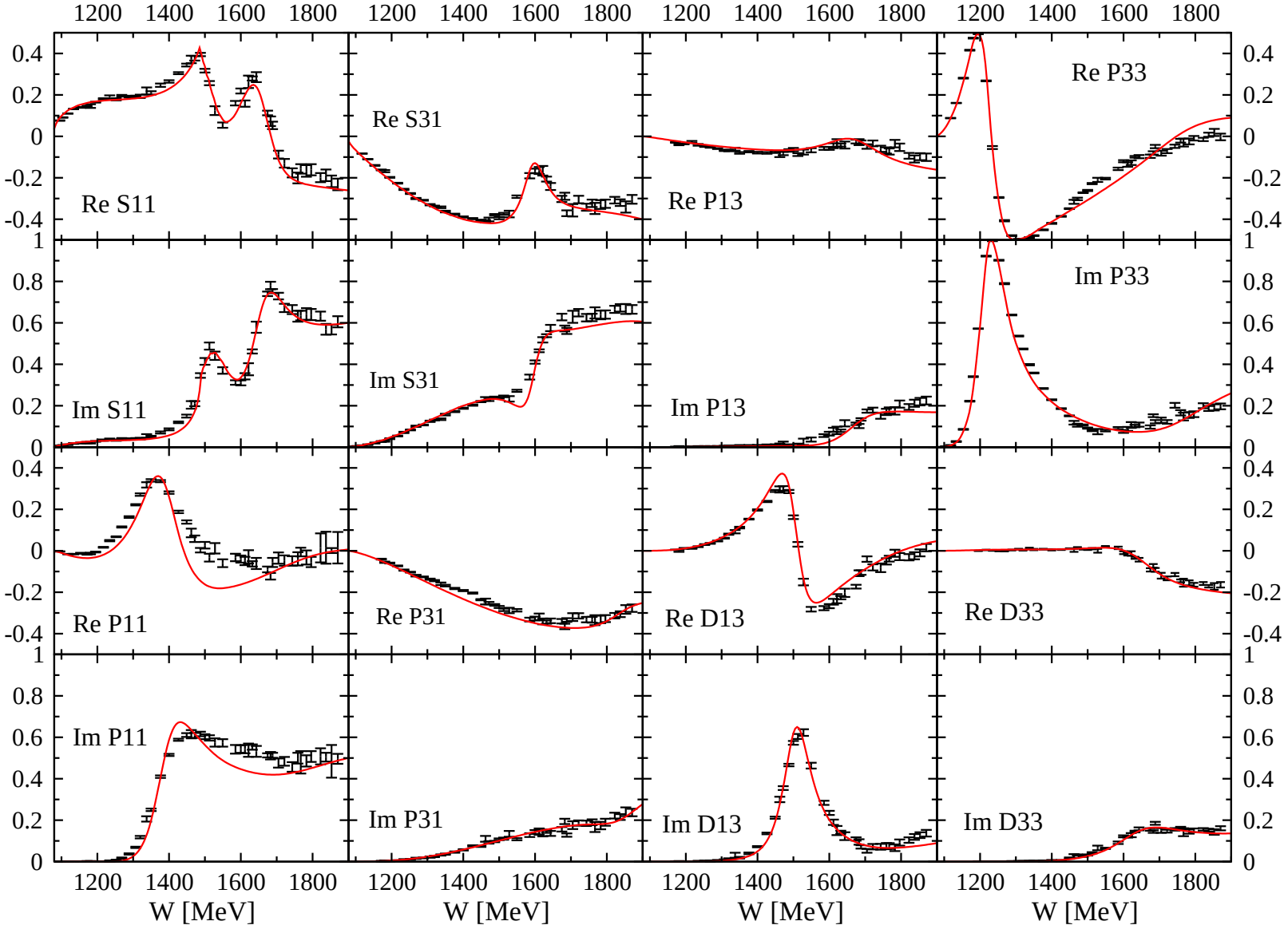
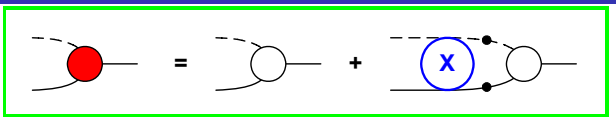


Photo-production amplitude H. Haberzettl, PRC56(1997)2041



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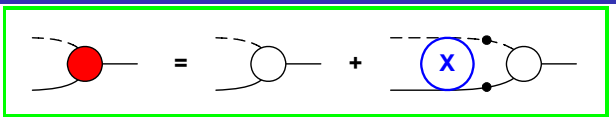

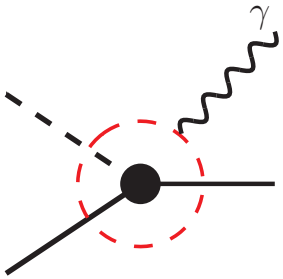

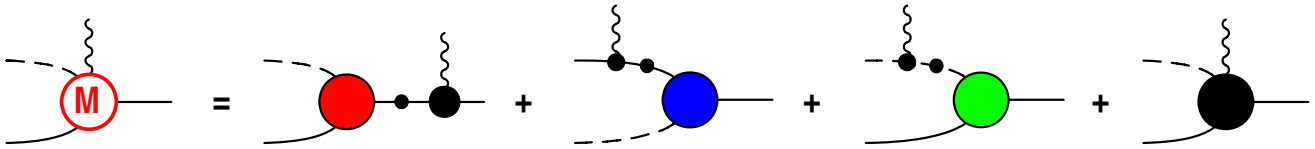

$$M^\mu = M_s^\mu + M_u^\mu + M_t^\mu + M_{\text{int}}^\mu$$

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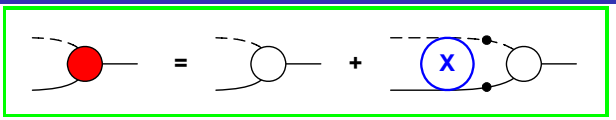

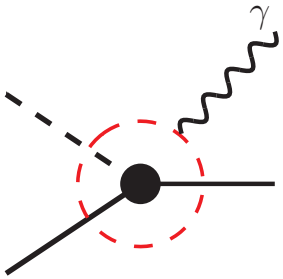

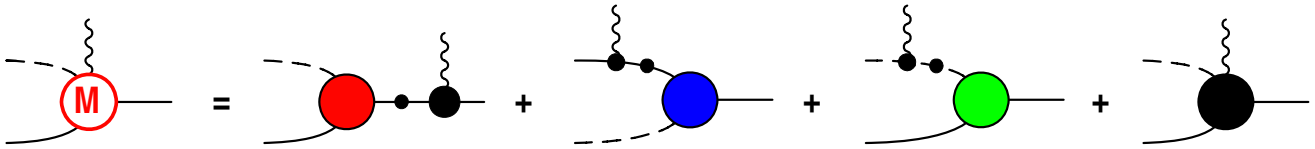

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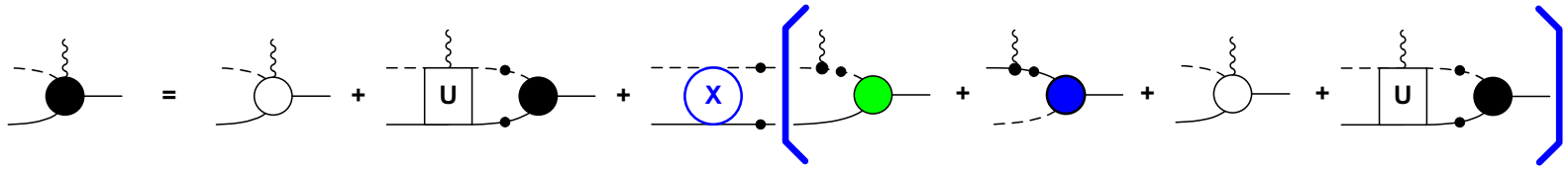

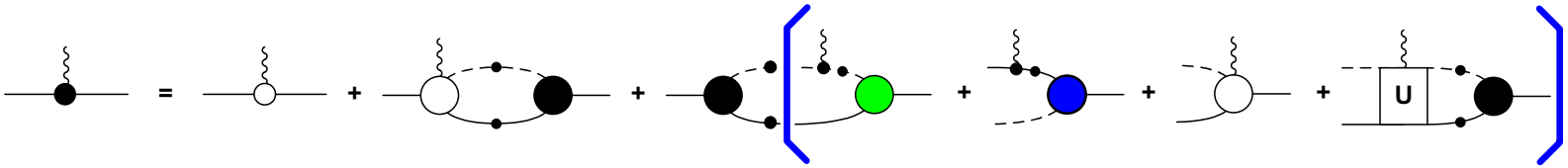

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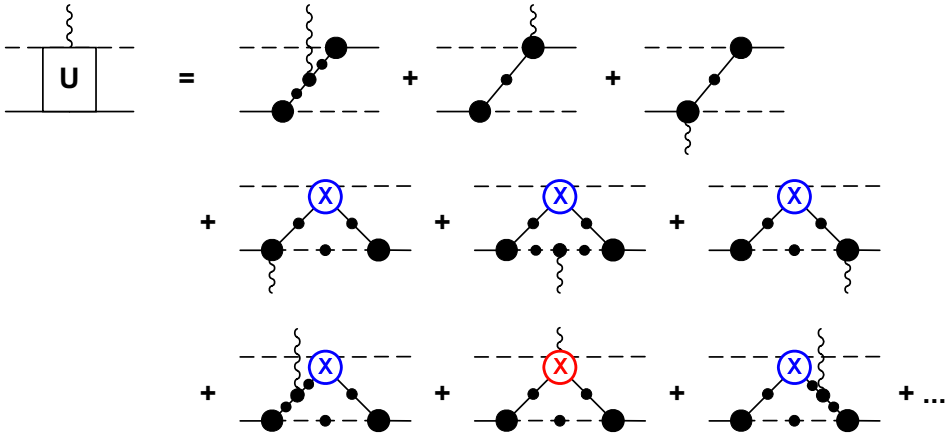

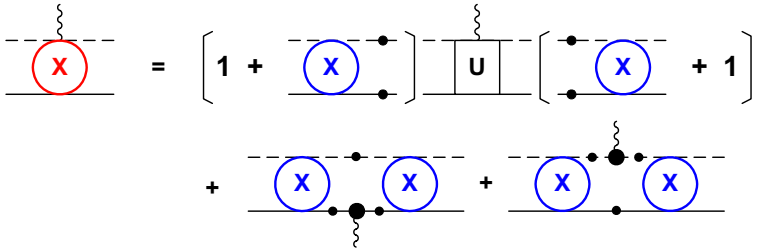

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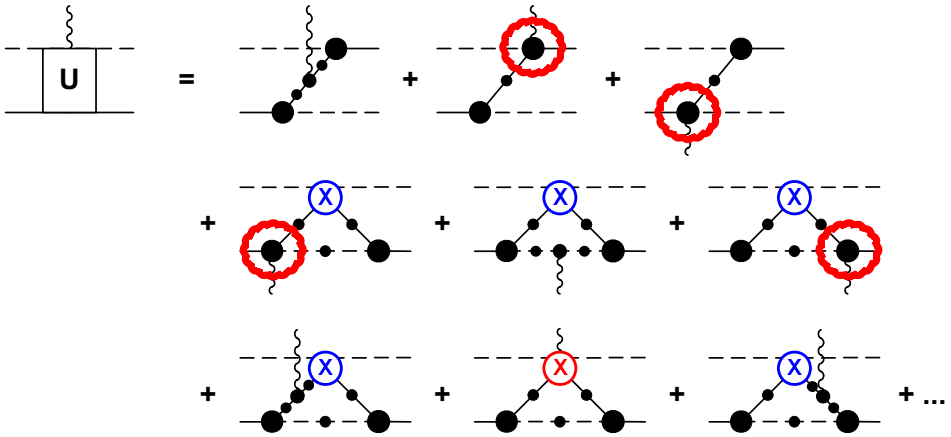

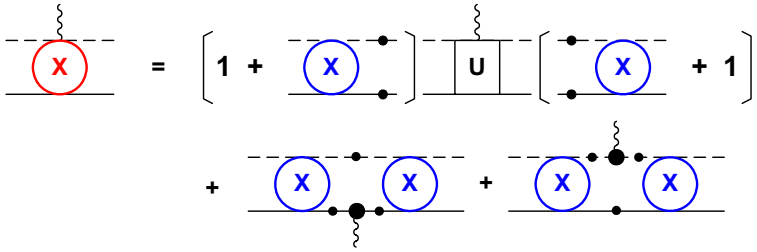

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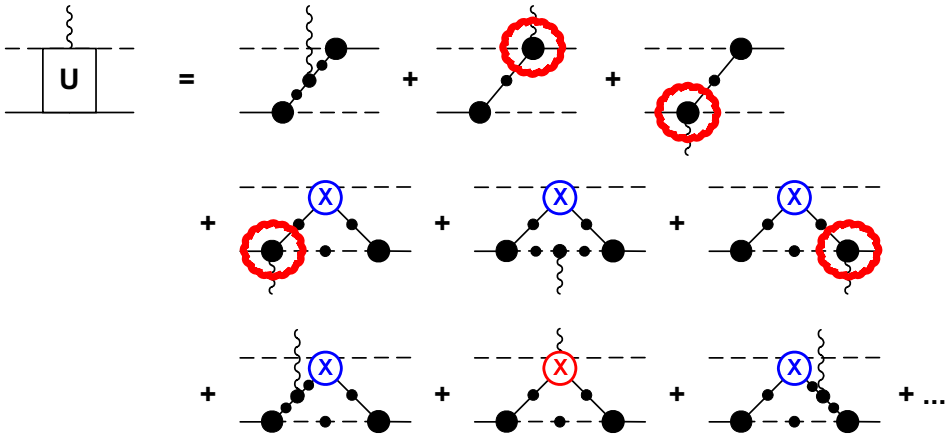
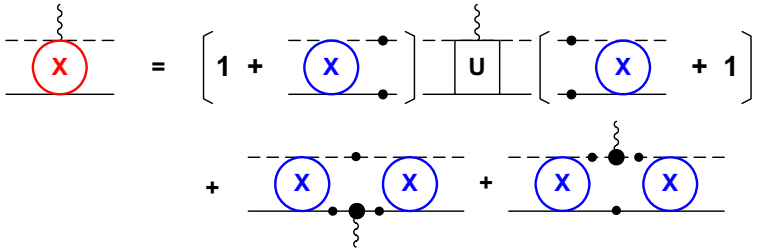


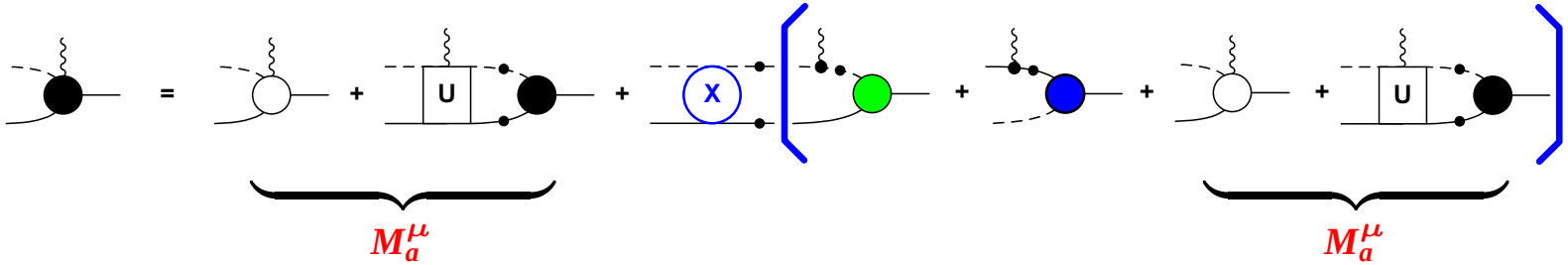


- Amplitude results from a highly non-linear equation
- Inner consistency requires generalized Ward-Takahashi identity

$$k_\mu M^\mu = -\,|\Gamma_s \tau\rangle\, S_{p+k}\, Q_i\, S_p^{-1} + S_{p'}^{-1}\, Q_f\, S_{p'-k}\, |\Gamma_u \tau\rangle + \Delta^{-1}_{p-p'+k}\, Q_\pi\, \Delta_{p-p'}\, |\Gamma_t \tau\rangle$$

Q: charge operator S: nucleon propagator Δ: pion propagator $|\Gamma\rangle$: dressed vertex

Practical strategy



$$\boxed{\boldsymbol{M}^\mu_{\rm int} \;=\; M^\mu_a \;+\; X\,G\,(M^\mu_t + M^\mu_u + M^\mu_a)}$$

Gauge invariance condition:

$$k_\mu M^\mu \;=\; -\,|\Gamma_s\tau\rangle\, S_{p+k}\, Q_i\, S_p^{-1} + S_{p'}^{-1}\, Q_f\, S_{p'-k}\,|\Gamma_u\tau\rangle + \Delta^{-1}_{p-p'+k}\, Q_\pi\, \Delta_{p-p'}\,|\Gamma_t\tau\rangle$$

$$\Updownarrow$$

$$k_\mu M^\mu_a \;=\; (1-U\,G)\,(-\,|\Gamma_s\rangle\, e_i + |\Gamma_u\rangle\, e_f + |\Gamma_t\rangle\, e_\pi) - k_\mu U\,G\,(M^\mu_{tL} + M^\mu_{uL})$$

Approximating M^μ_a under gauge invariance condition:

$$M^\mu_a \;=\; (1-U\,G)\,M^\mu_c - U\,G\,(M^\mu_{tL} + M^\mu_{uL}) + T^\mu$$

$$k_\mu M^\mu_c \;=\; -\,|\Gamma_s\rangle\, e_i + |\Gamma_u\rangle\, e_f + |\Gamma_t\rangle\, e_\pi, \qquad k_\mu T^\mu = 0$$

Interaction current:

$$\boxed{\boldsymbol{M}^\mu_{\rm int} \;=\; M^\mu_c + T^\mu + X\,G\,[(M^\mu_{uT} + M^\mu_{tT}) + T^\mu]}$$

Choosing the contact current M_c^μ

- Constraints: gauge invariance; contact term; crossing symmetry
- Introduce an auxiliary current C^μ:

$$C^\mu = -e_\pi \frac{(2q-k)^\mu}{t-q^2}\left(f_t - \hat{F}\right) - e_f \frac{(2p'-k)^\mu}{u-p'^2}\left(f_u - \hat{F}\right) - e_i \frac{(2p+k)^\mu}{s-p^2}\left(f_s - \hat{F}\right)$$

$$\hat{F} = 1 - \hat{h}\left(1-\delta_s f_s\right)\left(1-\delta_u f_u\right)\left(1-\delta_t f_t\right)$$

k, p, q, p': 4-momenta for incoming γ, N & outgoing π, N $\hat{h}$: fit parameter

- The contact current M_c^μ can be written as

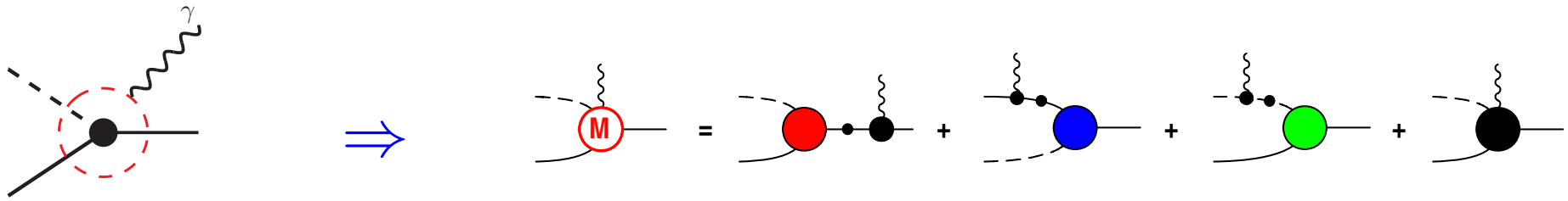
$$M_c^\mu = g_\pi \gamma_5 \left\{ \left[\lambda + (1-\lambda)\frac{\not{q} - \beta \not{k}}{m'+m}\right] C^\mu - (1-\lambda)\frac{\gamma^\mu}{m'+m}\left[e_\pi f_t - \beta k_\rho C^\rho\right] \right\}$$

β: fit parameter

- Check gauge invariance: M_c^μ satisfies

$$k_\mu M_c^\mu = -\left|\Gamma_s\right\rangle e_i + \left|\Gamma_u\right\rangle e_f + \left|\Gamma_t\right\rangle e_\pi$$

Reaction theory: all together



Full photo-production current:

$$M^\mu = M_s^\mu + M_u^\mu + M_t^\mu + \boldsymbol{M}_{\rm int}^\mu$$

$$\boldsymbol{M}_{\rm int}^\mu = M_c^\mu + T^\mu + X\,G\,[(M_{uT}^\mu + M_{tT}^\mu) + T^\mu]$$

M^μ satisfies the generalized Ward-Takahashi identity (gauge invariant)

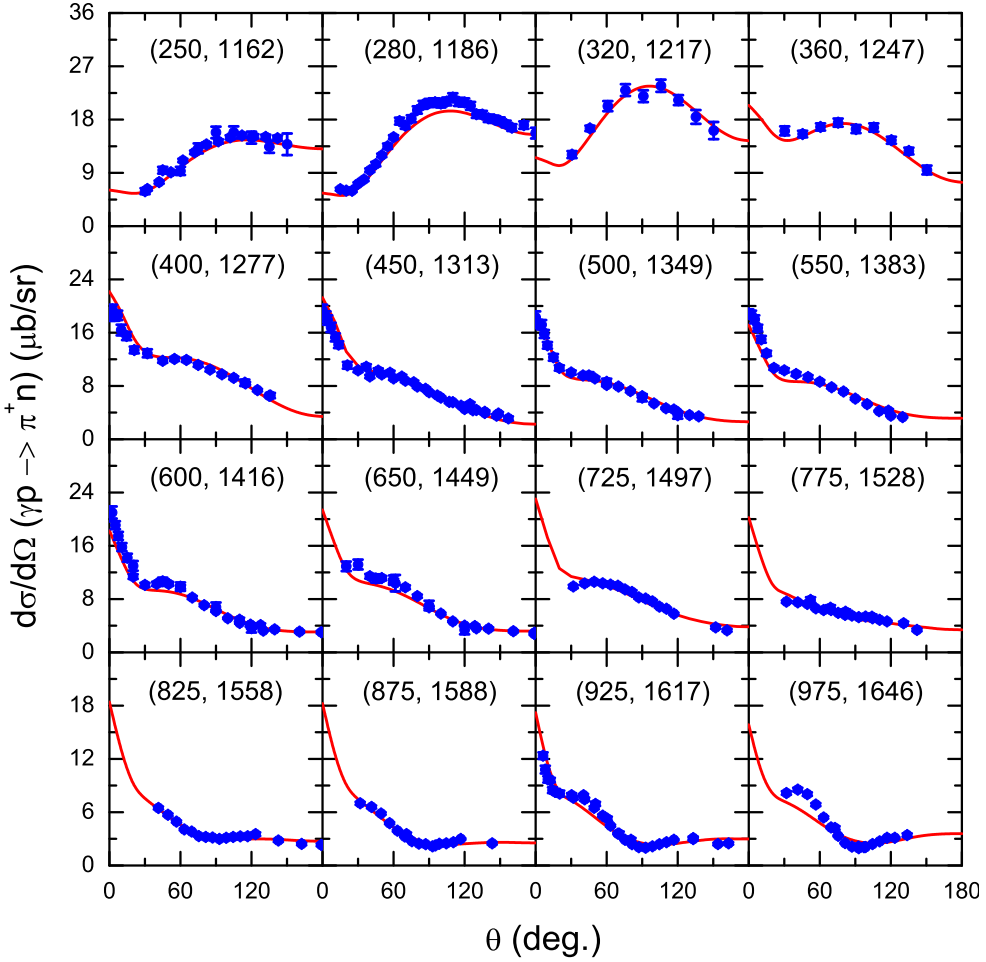
$$k_\mu M^\mu = -\,|\Gamma_s \tau\rangle\, S_{p+k}\, Q_i\, S_p^{-1} + S_{p'}^{-1}\, Q_f\, S_{p'-k}\, |\Gamma_u \tau\rangle + \Delta_{p-p'+k}^{-1}\, Q_\pi\, \Delta_{p-p'}\, |\Gamma_t \tau\rangle$$

T^μ: undetermined transverse contact current (set to be zero in this work)

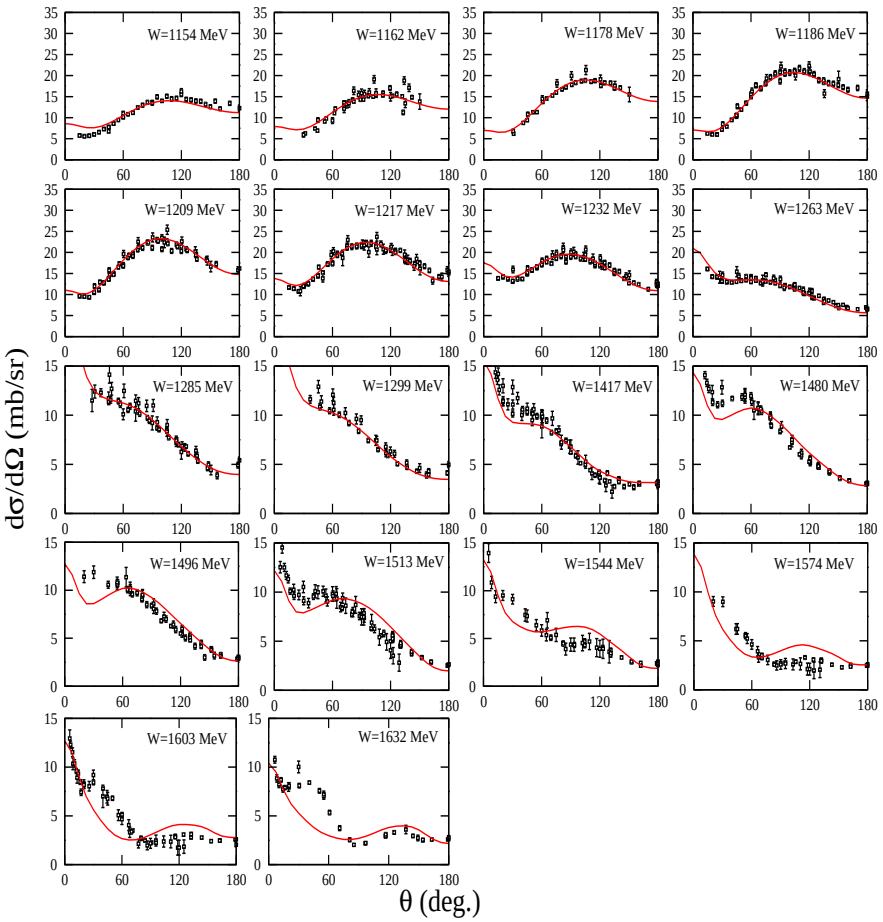
X: non-polar part of hadronic scattering (Jülich coupled-channel πN model)

Results: $d\sigma/d\Omega$ for $\gamma p \rightarrow \pi^+ n$

Jülich-Georgia



EBAC



Results: Σ_γ for $\gamma\, p \rightarrow \pi^+ n$

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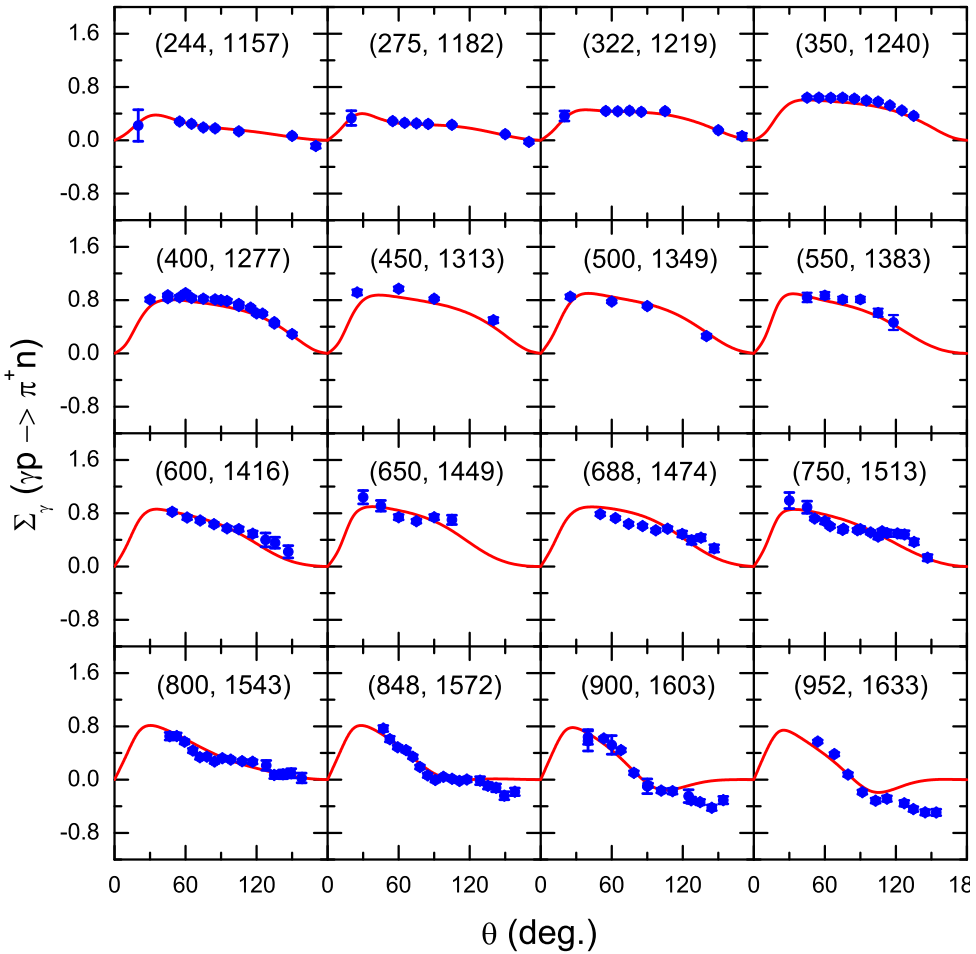

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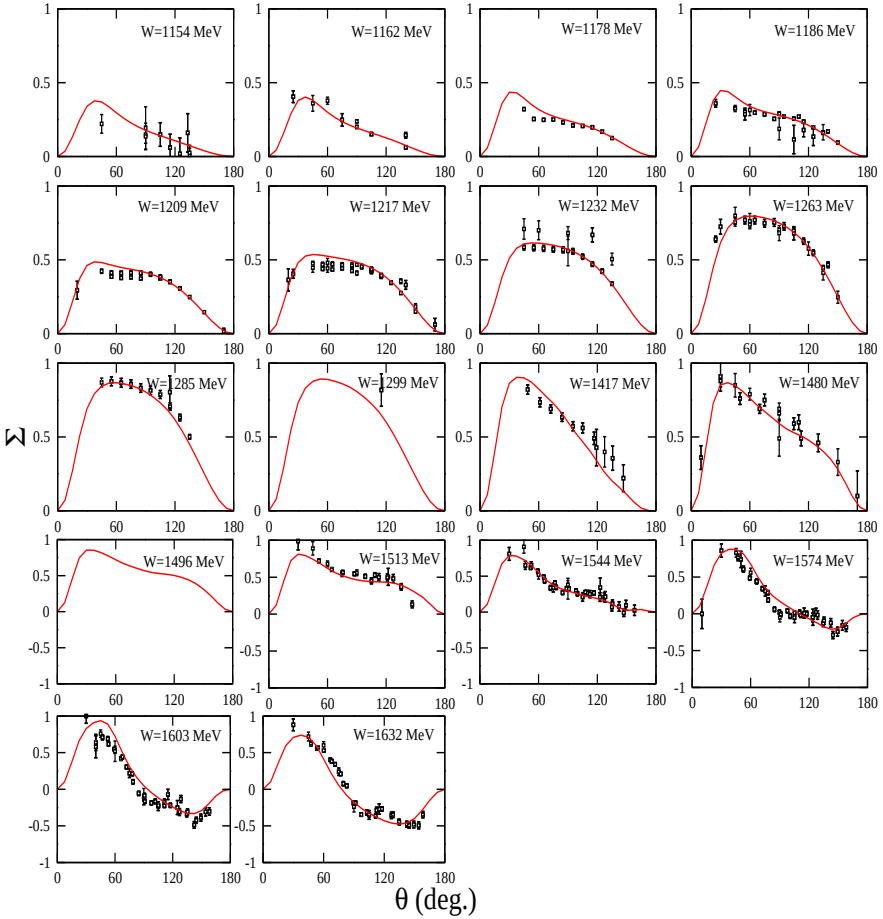
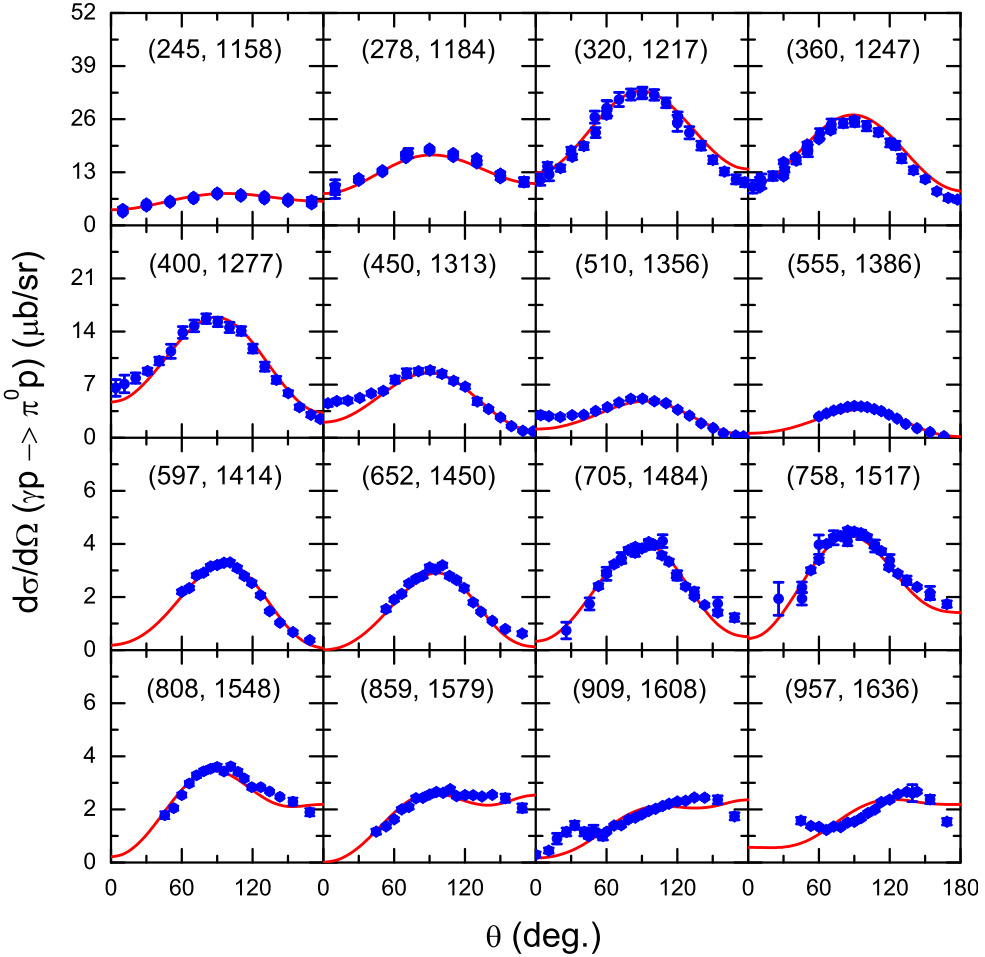


Results: $d\sigma/d\Omega$ for $\gamma p \rightarrow \pi^0 p$

Jülich-Georgia



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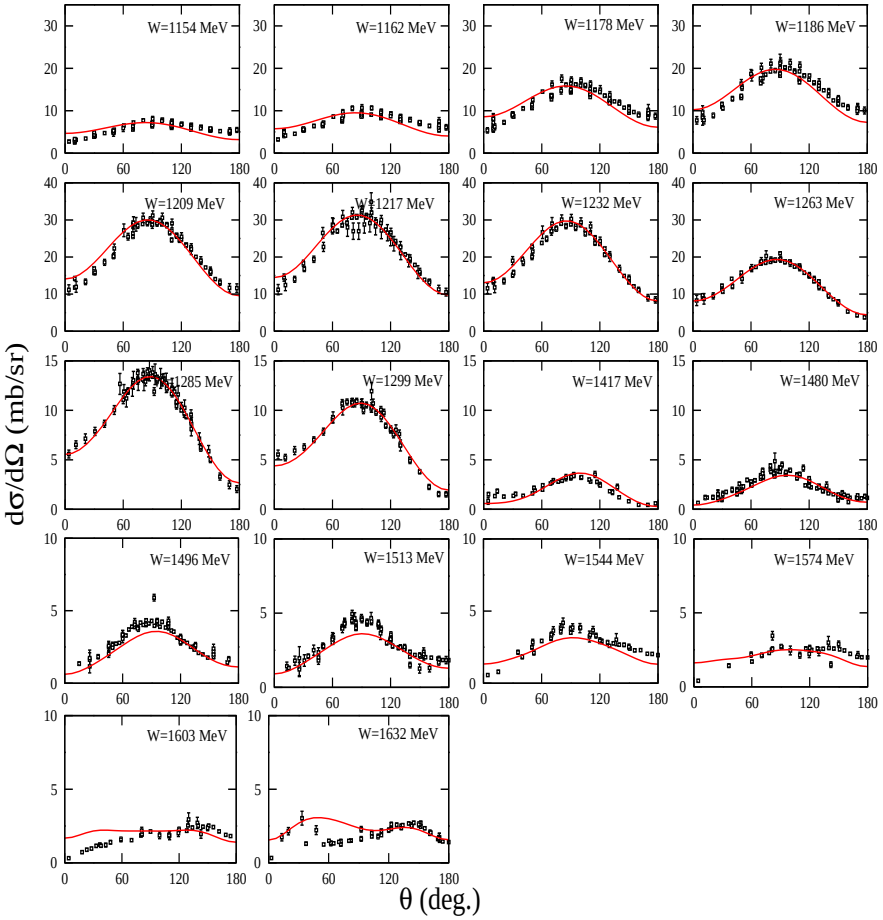

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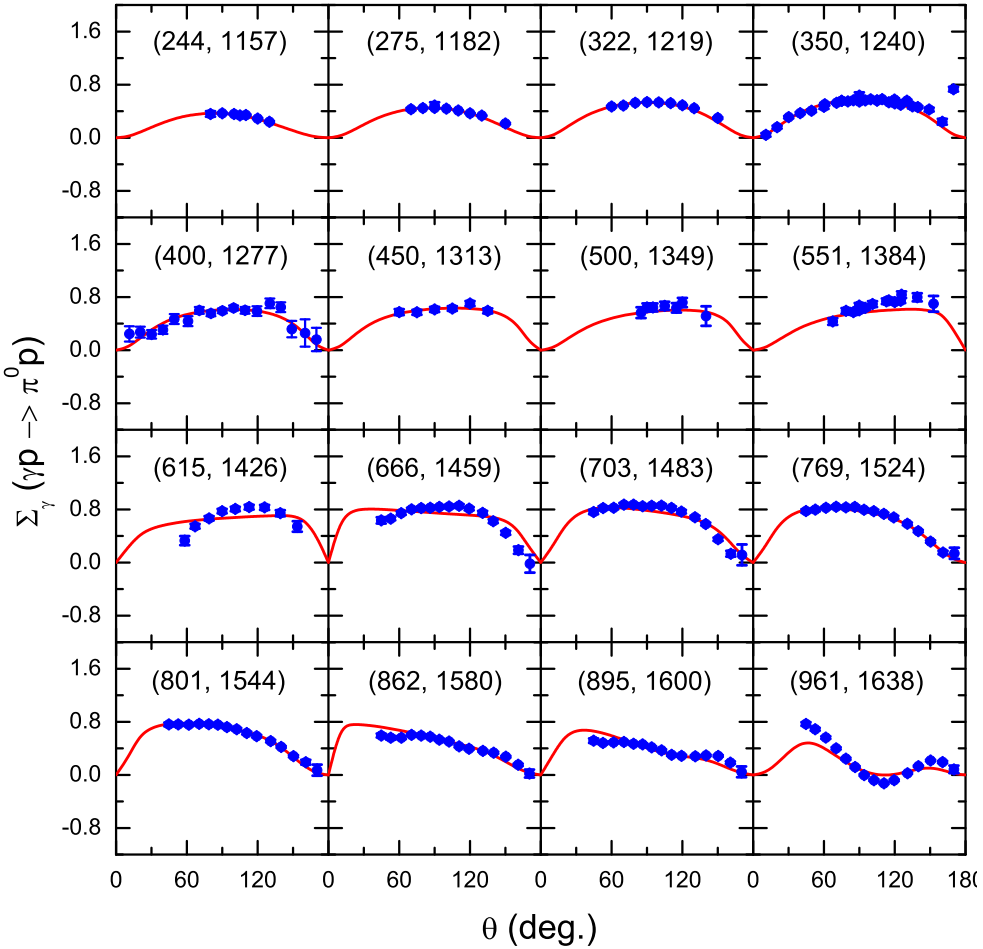

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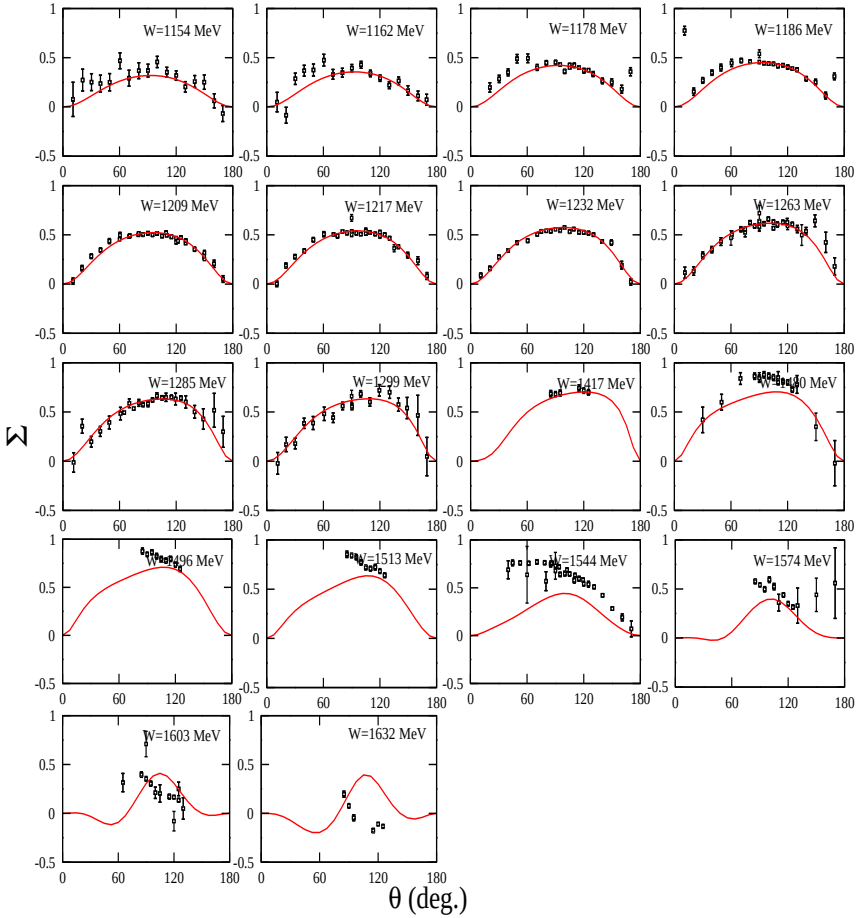
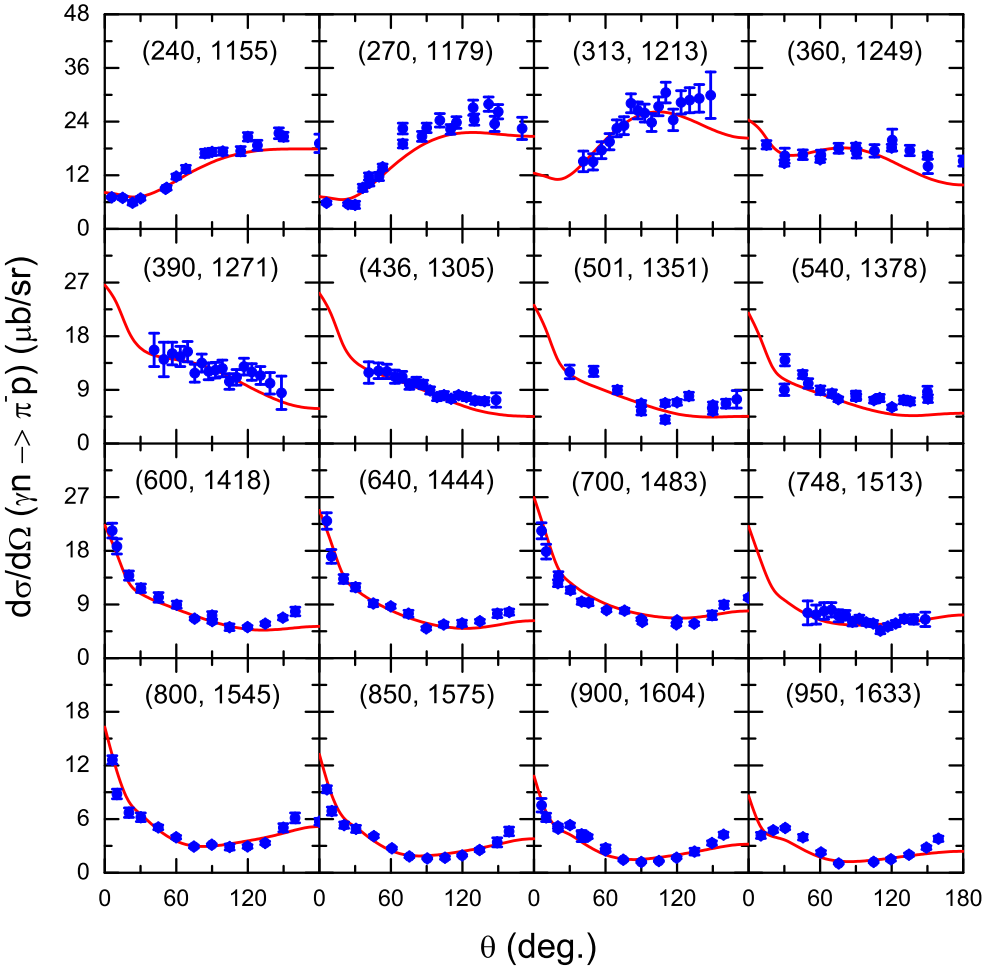


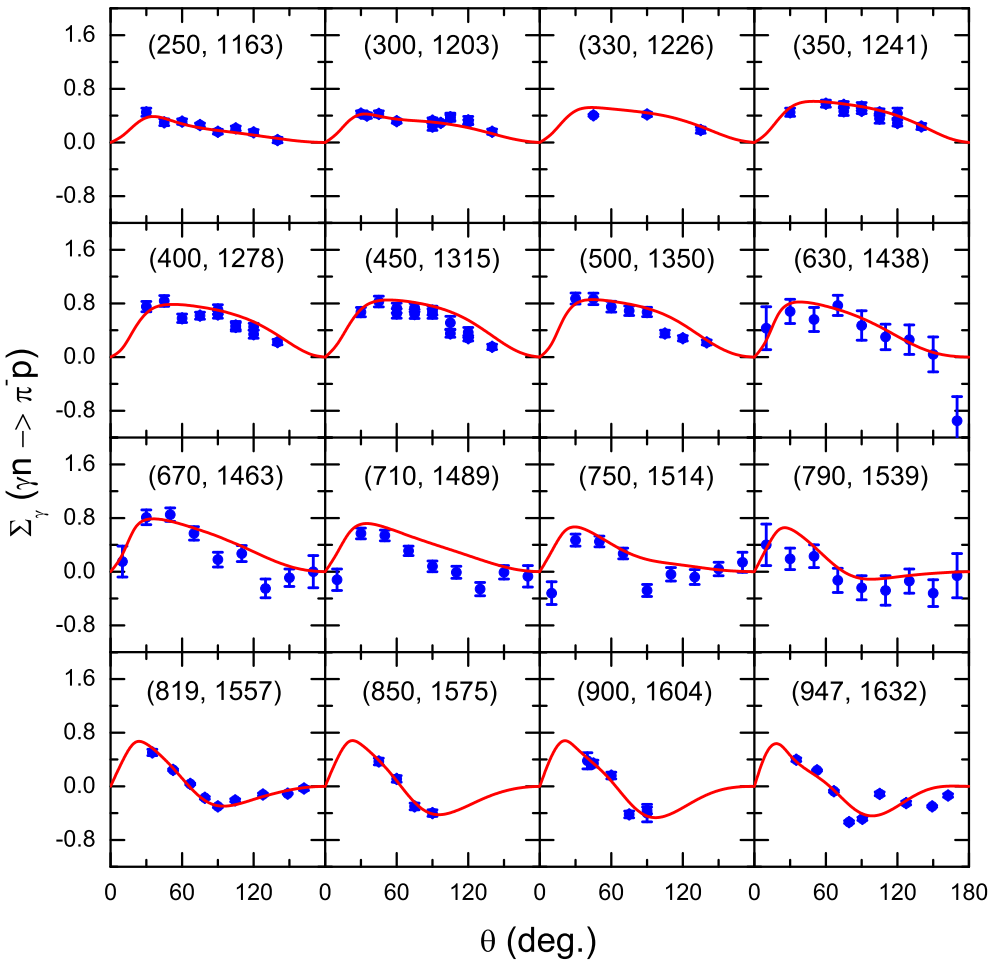
Results: $d\sigma/d\Omega$ & Σ_γ for $\gamma\, n \rightarrow \pi^- p$

Jülich-Georgia



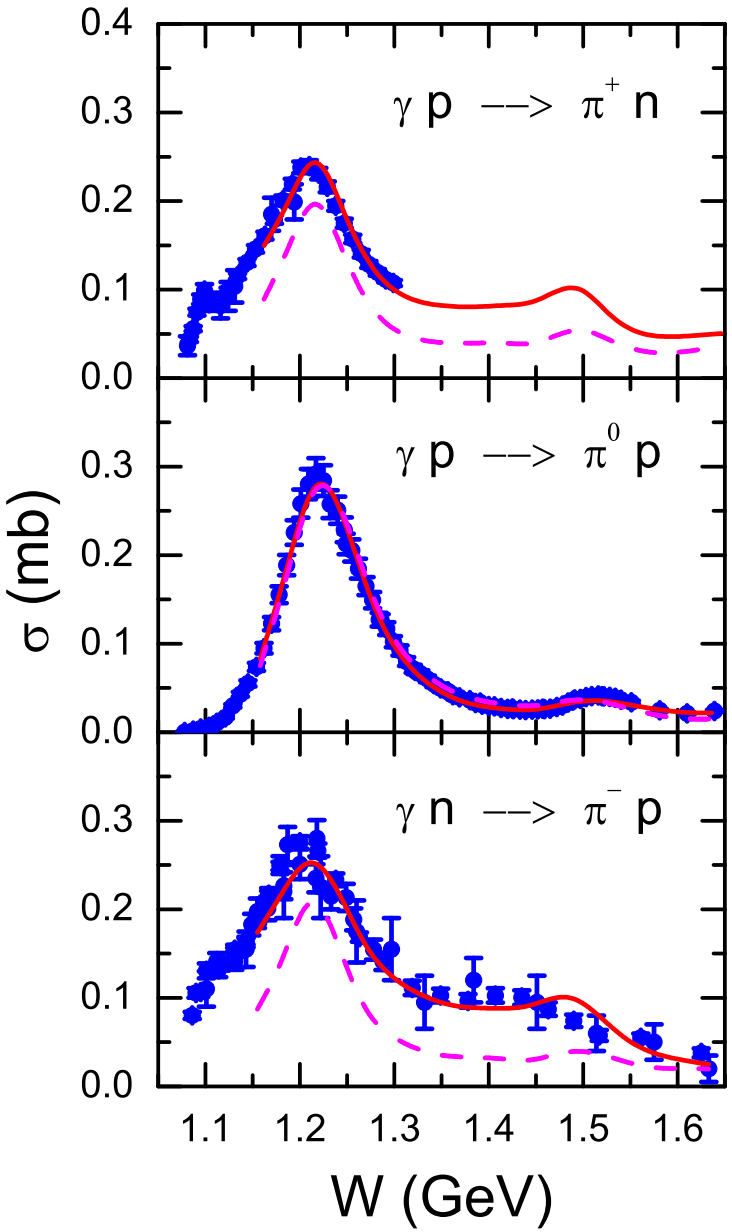
Differential cross section for $\gamma\, n \rightarrow \pi^- p$

Jülich-Georgia



Photon spin asymmetry for $\gamma\, n \rightarrow \pi^- p$

Results: $\gamma N \longrightarrow \pi N$ total cross sections



Full results

Contact term off

data not included in the fit

Resonance & background

- Splitting $T = T^P + T^{NP}$ arbitrary and highly model-dependent
  e.g. Jülich model and EBAC model both get the data but they have quite different T^{NP}
- Identifying T^{NP} as T^{BG} problematic

$$T = T^P + T^{NP}$$

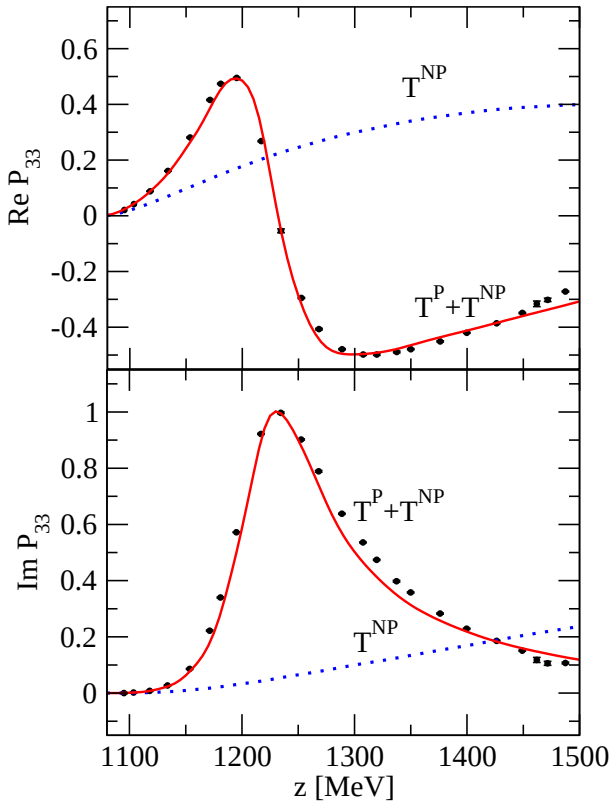
$$T = \frac{a_{-1}}{z - z_0} + a_0 + \mathcal{O}(z - z_0)$$

$$a_0 = a_0^P + T^{NP}$$

$$T = T^R + T^{BG}$$

$$T^R = \frac{a_{-1}}{z - z_0}$$

$$T^{BG} = T - T^R$$



- Pole position & residue are less model-dependent
- T^R & T^{BG} more meaningful than T^P & T^{NP} to compare in various models

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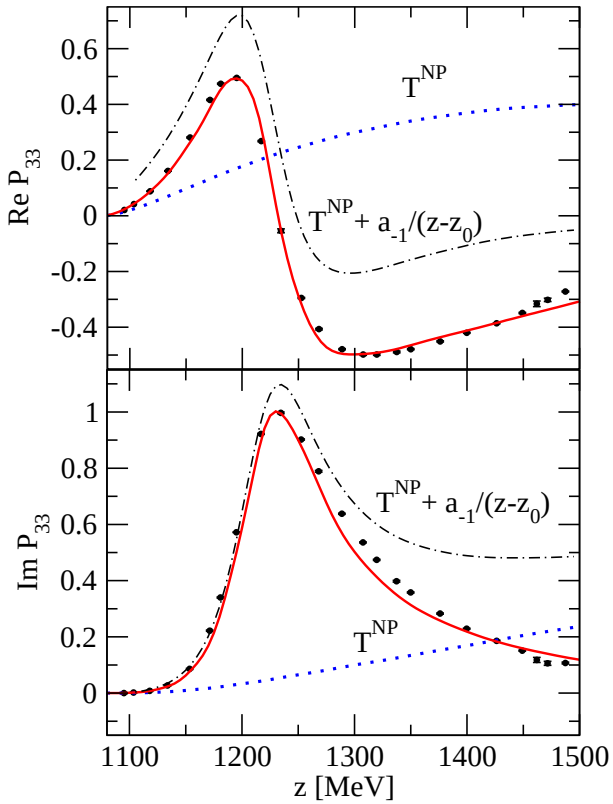
$$T = T^R + T^{BG}$$

$$T^R = \frac{a_{-1}}{z - z_0}$$

$$T^{BG} = T - T^R$$



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$$T = T^P + T^{NP}$$

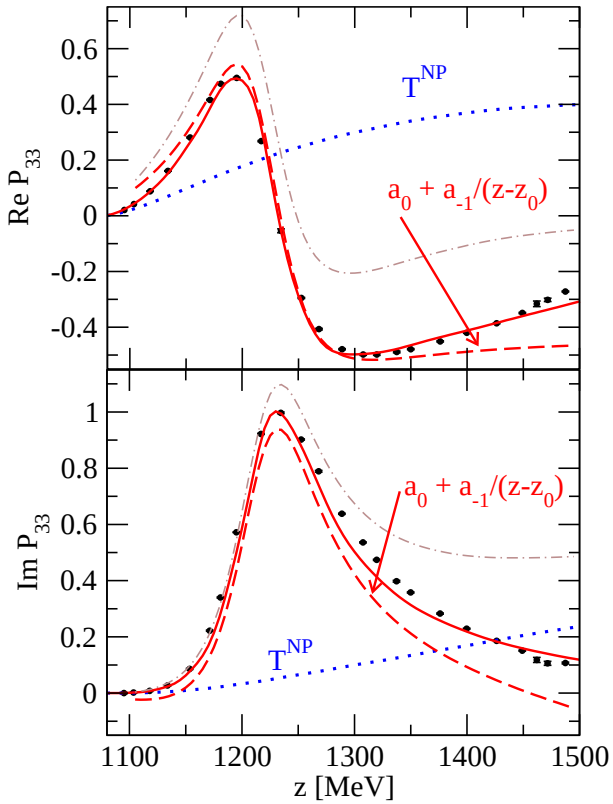
$$T = \frac{a_{-1}}{z - z_0} + a_0 + \mathcal{O}(z - z_0)$$

$$a_0 = a_0^P + T^{NP}$$

$$T = T^R + T^{BG}$$

$$T^R = \frac{a_{-1}}{z - z_0}$$

$$T^{BG} = T - T^R$$



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Poles & residues

Table: Effective electromagnetic couplings g^γ in helicity basis (PRELIMINARY). The 1st & 2nd lines for isospin $1/2$ resonance correspond to results for p & n, respectively.

	Pole position [MeV]	$g^\gamma_{1/2}$ [MeV$^{1/2}$]	$g^\gamma_{3/2}$ [MeV$^{1/2}$]
$P_{33}(1232)$	$1217 - 45\,\mathrm{i}$	$0.42 - 0.11\,\mathrm{i}$	$4.95 + 1.53\,\mathrm{i}$
$P_{11}(1440)$	$1385 - 72\,\mathrm{i}$	$-0.07 - 0.26\,\mathrm{i}$	
		$-0.92 + 0.08\,\mathrm{i}$	
$D_{13}(1520)$	$1503 - 47\,\mathrm{i}$	$-3.55 + 2.14\,\mathrm{i}$	$2.80 - 1.34\,\mathrm{i}$
		$1.99 - 1.84\,\mathrm{i}$	$-4.00 + 1.68\,\mathrm{i}$
$S_{11}(1535)$	$1520 - 64\,\mathrm{i}$	$-1.52 + 1.87\,\mathrm{i}$	
		$4.05 - 2.01\,\mathrm{i}$	
$S_{31}(1620)$	$1592 - 37\,\mathrm{i}$	$-0.43 - 0.19\,\mathrm{i}$	
$S_{11}(1650)$	$1666 - 70\,\mathrm{i}$	$3.31 + 2.67\,\mathrm{i}$	
		$-1.22 - 2.50\,\mathrm{i}$	
$D_{33}(1700)$	$1638 - 122\,\mathrm{i}$	$0.37 - 8.98\,\mathrm{i}$	$-3.91 + 0.81\,\mathrm{i}$
$P_{13}(1720)$	$1665 - 101\,\mathrm{i}$	$0.11 - 5.41\,\mathrm{i}$	$0.54 + 1.55\,\mathrm{i}$
		$-0.51 + 2.71\,\mathrm{i}$	$0.28 - 3.23\,\mathrm{i}$
$P_{31}(1910)$	$1833 - 110\,\mathrm{i}$	$59.29 - 31.11\,\mathrm{i}$	

$$g^\gamma = a^\gamma_{-1}/g^\pi \qquad g^\pi = \sqrt{a^\pi_{-1}}$$

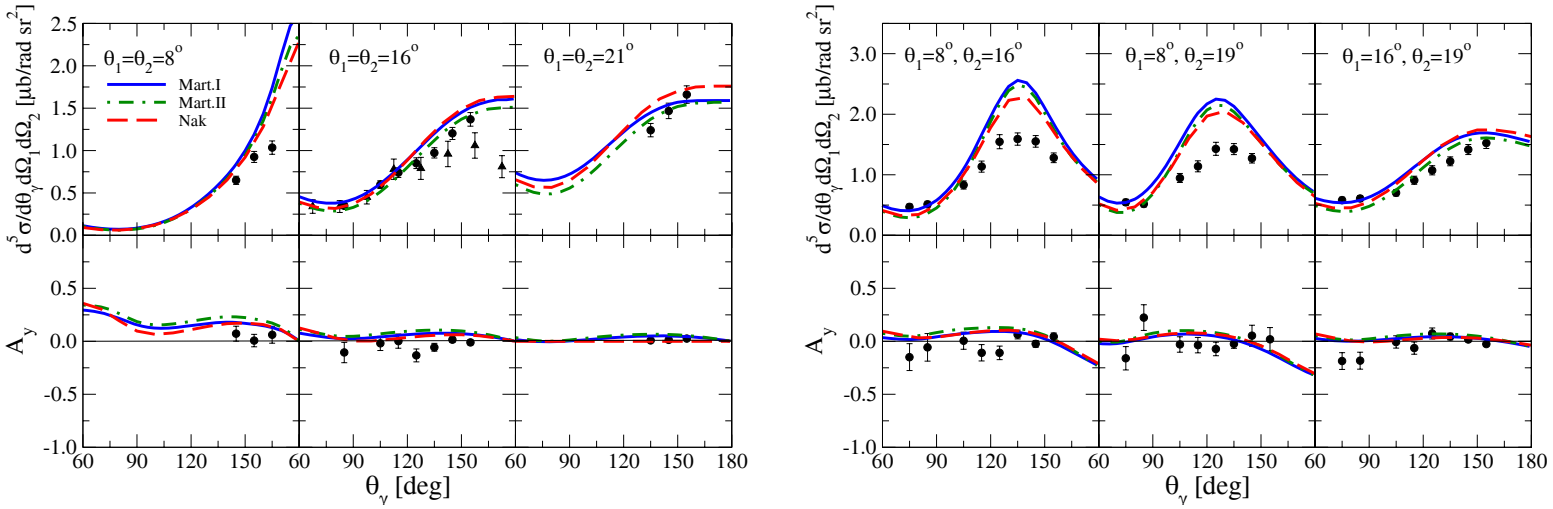
Summary & perspectives

- Reaction theory is based on Jülich dynamical coupled-channel model which describes πN scattering successfully
- Full photoproduction amplitudes satisfy the generalized Ward-Takahashi identity and thus are gauge invariant
- Both the differential cross sections and photon spin asymmetries for π photoproduction are described quite well up to $\sqrt{s} = 1.65$ GeV
- Effective electromagnetic couplings (preliminary) are extracted by analytic continuation of the full amplitudes to un-physical Riemann sheet
- Resonance and background contributions will be studied
- Future plan:
  - η- and K-photoproduction
  - Electro-production

Importance of gauge invariance: $p\,p \rightarrow p\,p\,\gamma$

None of the existing models can describe high-precision KVI data for coplanar geometries involving small p scattering angles.

Lines: Martinus, Scholten, Tjon, PRC 58, 686 (1998); PRC 56, 2945 (1997); Herrmann, Nakayama, Scholten, Arellano, NPA 582, 568 (1995)



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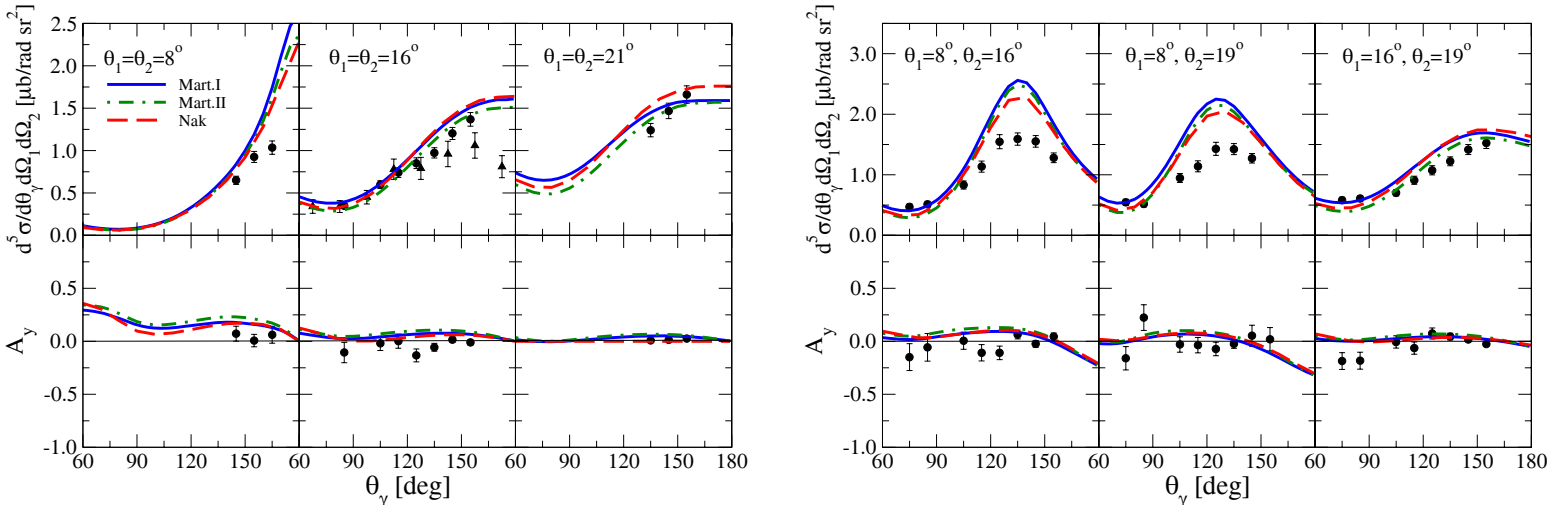
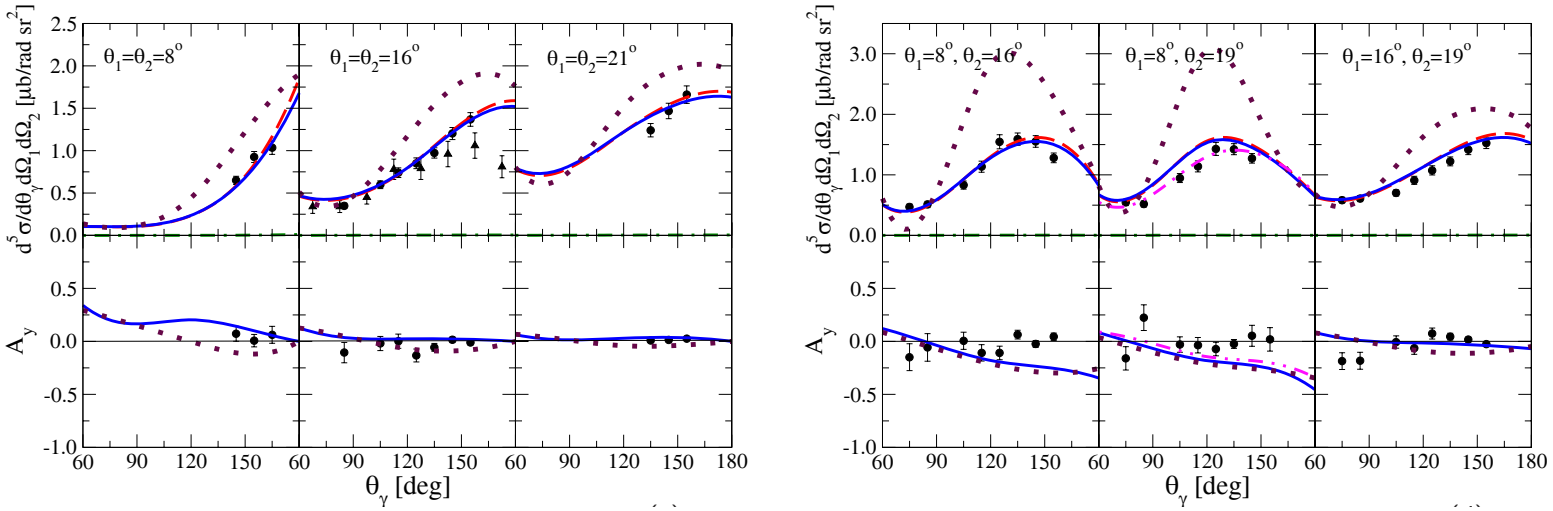
Lines: Martinus, Scholten, Tjon, PRC 58, 686 (1998); PRC 56, 2945 (1997); Herrmann, Nakayama, Scholten, Arellano, NPA 582, 568 (1995)



Construct the contact current $\to$ full amplitude obeys WTI (gauge invariant) [K. Nakayama & H. Haberzettl, PRC 80, 051001 (2009)]



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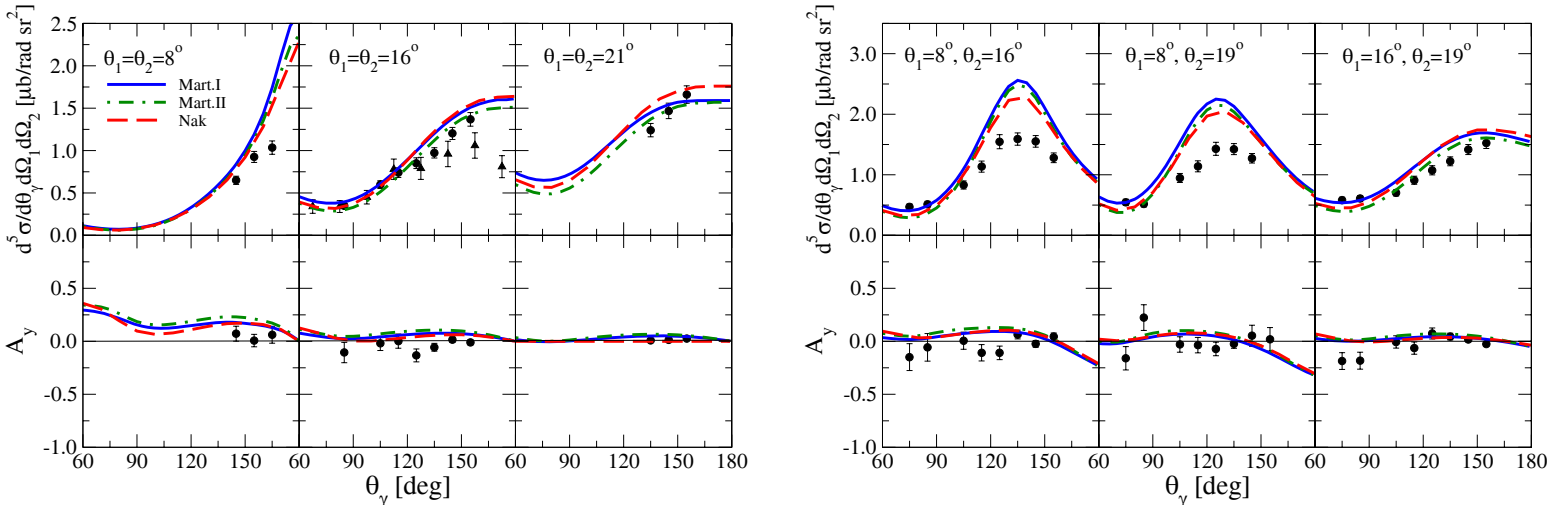

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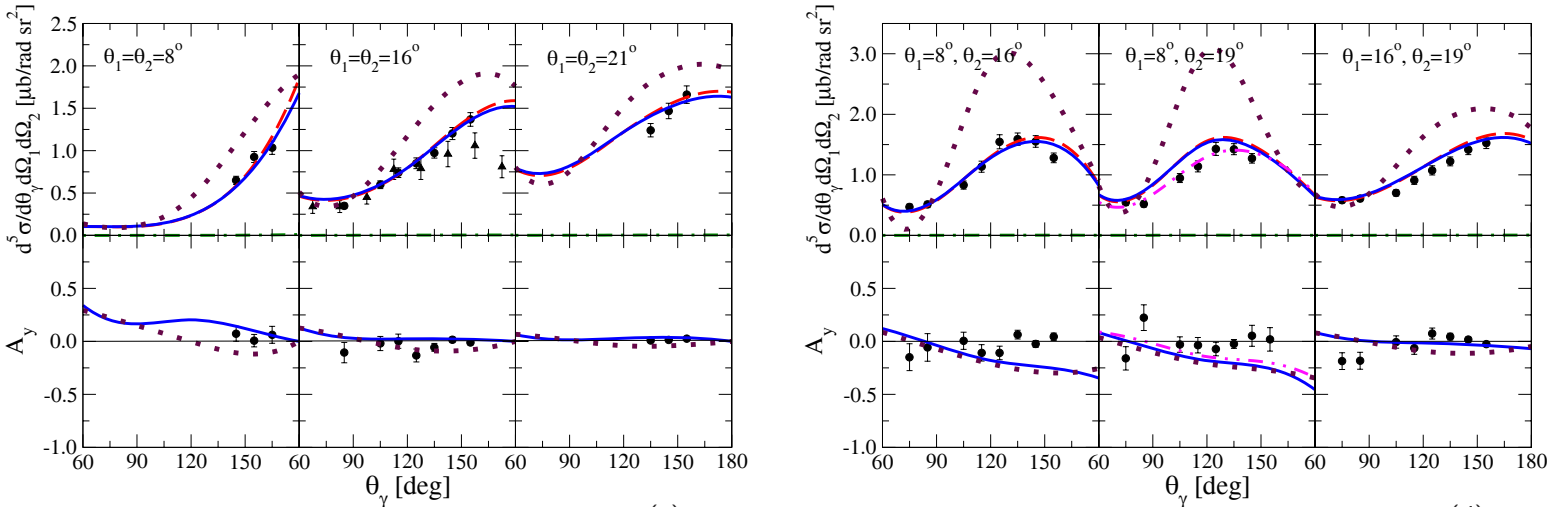

It is important to properly take into account the interaction current for NN bremsstrahlung reaction!